REAL OPTION VALUE

CHAPTER 14 B INCENTIVE OPTIONS

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Incentive options can be viewed using the toolkit implicit in previous chapters of real payoff diagrams, entry and exit options, and perpetual American puts and calls. Incentive options may be granted (or required by) governments to encourage early investment in "desirable" projects such as renewable energy facilities, infrastructure investments like roads, bridges and other transportation, and in general public-private partnerships (PPP) governing new facilities like schools, hospitals, and recreation areas.

These incentive options are classified as (i) proportional revenue (or price and/or quantity) subsidies, where the market price and/or the quantity of production is uncertain or low, but the subsidy is proportional to the quantity produced (ii) supplementary revenue (or price and/or quantity) subsidies, where the market price and/or the quantity of production and/or the exogenous subsidy is uncertain (iii) revenue floors and ceilings, where the subsidy is related over time to the actual quantities produced or market prices. Examples of (i) are so-called Feed-in-Tariffs "FiT" which are fixed amount subsidies per unit production, (ii) renewable "green" certificates, which have an uncertain value but are usually allocated per unit of production, and (iii) government minimum revenue guarantees, sometimes accompanied by maximum revenue ceilings.

In addition, governments provide incentives in the form of free or at low cost assets (sport stadiums), protection through tariffs, quotas or security, in order to encourage "desirable" activities, or investment cost reliefs, consisting of direct grants and soft loans, tax credits or excess depreciation. Although not directly considered herein, some of these incentives can be evaluated in terms of the real option value compared to that paid to the government (taxes, concession and user fees and royalties) weighted against the immediate or eventual cost for the government. It is interesting to study the effect of incentives on the real option value, and on the threshold that justifies immediate investment, as price, quantity and subsidies change. Who gets/gives what, when, how, and why are almost always critical considerations in incentive options.

14.1 Subsidies & Revenue Limits

The real American collar option for a certain asset confines the effective price within specified floor (lower) and ceiling (upper) limits. Acting as a risk moderator, the collar offers protection against the adversity from extreme falls in the output price or rises in the procurement price while simultaneously extracting some incremental value from favorable prices. Consequently, the upside gains partially compensate the downside losses. Unlike financial options, real American perpetuities on specific projects are currently not obtainable from the market, but governments may be agreeable to grant and underwrite price limits in certain circumstances. The pursuance of an energy diversity goal may motivate governments to enact a policy that subsidizes renewable energy investors by guaranteeing a fixed price in the form of a contract-for-differences deal. Similarly, foreign investors are induced to locate in countries whose governments grant subsidized or preferential procurement prices for raw materials or energy. The role of these subsidies is to raise the investment option value and to reduce the investment threshold, which not only render an investment more attractive but also hasten its exercise.

In a real option framework, there are several articles on the effect of a subsidy on the investment value and policy. Boomsma et al. (2012) evaluate energy subsidies. Adkins and Paxson (2015, 2016A) consider permanent and retractable subsidies as do Boomsma and Linnerud (2015), but not revenue ceilings. Takashima et al. (2010) design a PPP deal involving government debt participation that incorporates a floor on the future maximum loss level, where the concessionaire has the right to sell back the project to the government whenever adverse conditions emerge. Armada et al. (2012) investigate a subsidy in the form of a perpetual put option on the output price with protection against adverse price movements.

Adkins and Paxson (2016B) consider perpetual collar options in PPPs. From a general model, separate price floor subsidies and price ceilings are specific examples of general collar options imposed on the active project value. A price collar option contributes both positively and negatively to the active project value, and also to the real option value of an opportunity to invest in such a project.

14.2 Real Collar Option for an ACTIVE Asset

Suppose a firm in a monopolistic setting confronts a single source of uncertainty due to revenue variability, and ignoring operating costs and taxes, the opportunity to invest in an irretrievable project at cost K depends solely on the revenue evolution, which is specified by the geometric Brownian motion process:

$$\mathrm{dR} = \alpha R \mathrm{d}t + \sigma R \mathrm{d}W \tag{1}$$

where α denotes the expected revenue risk-neutral drift, σ the revenue volatility, and $\mathrm{d}W$ an increment of the standard Wiener process.

A collar option is designed to confine the revenue for an active project to a tailored range, by restricting its value for the project owner to lie between a floor R_L and a ceiling R_H, where typically the government granting the concession guarantees the floor and receives all R over the ceiling. Whenever R falls below the floor, the received R is assigned the value R_L, and whenever it exceeds the ceiling, it is assigned the value R_H. Protection against downside losses for the owner are mitigated in part by sacrificing upside gains. The value of a project with a perpetual net revenue R is R/δ, where δ is the yield for a similar risky project. Using contingent claims analysis, for an active project with a collar, the revenue accruing to the owner is given by $\pi_C(R) = \min\{\max\{R_L, R\} R_H\}$ and its value V_C is described by the risk-neutral valuation relationship:

$$\tfrac{1}{2}\sigma^2 R^2 \frac{\partial^2 V_C}{\partial R^2} + (r-\delta) R \frac{\partial V_C}{\partial r} - rV_C + \pi_C(R) = 0 . \tag{2}$$

where $r > \alpha$ denotes the risk-free interest rate and $\delta = r - \alpha$ the rate of return shortfall, or net asset yield. The generic solution to the option part of (2) is:

$$V(R) = A_1 R^{\beta_1} + A_2 R^{\beta_2} \tag{3}$$

where A_1, A_2 are to be determined generic constants and β_1, β_2 are, respectively, the positive and negative roots of the fundamental quadratic equation, which are:

$$\beta_1, \beta_2 = \left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{4}$$

In (3), if $A_2 = 0$ then V is a continuously increasing function of R and represents an American perpetual call option, Samuelson (1965), while if $A_1 = 0$ then it is a decreasing function and represents an American put option, Merton (1973).

The subscript C denotes the with-collar arrangement, and the valuation function for the owner of an active project is:

$$V_C\left(R\right)=\begin{cases}\dfrac{R_L}{r}+A_{C11}R^{\beta_1} & \text{for R} < R_L\\ \dfrac{R}{\delta}+A_{C21}R^{\beta_1}+A_{C22}R^{\beta_2} & \text{for R}_L \le R < R_H\\ \dfrac{R_H}{r}+A_{C32}R^{\beta_2} & \text{for R}_H \le R.\end{cases} \tag{5}$$

In (5), the first numerical subscript for a coefficient denotes the regime $\{1=I,2=II,3=III\}$, while the second denotes a call if 1 or a put if 2. The coefficients A_{C11}, A_{C22} are expected to be positive because the owner holds the options and a switch is beneficial. In contrast, the A_{C21}, A_{C32} are expected to be negative because the owner has "written" the options and is being penalized by the switch. The real collar is composed of four call and put options. A switch in either direction between Regimes I and II occurs when $R=R_L$. It is optimal provided the value-matching relationship:

$$\frac{R_L}{r}+A_{C11}R^{\beta_1}=\frac{R}{\delta}+A_{C21}R^{\beta_1}+A_{C22}R^{\beta_2} \tag{6}$$

and its smooth-pasting condition expressed as:

$$\beta_1 A_{C11}R^{\beta_1}=\frac{R}{\delta}+\beta_1 A_{C21}R^{\beta_1}+\beta_2 A_{C22}R^{\beta_2} \tag{7}$$

both hold when evaluated at $R=R_L$. Similarly, a switch in either direction between Regimes II and III occurs when $R=R_H$. It is optimal provided the value-matching relationship:

$$\frac{R}{\delta}+A_{C21}R^{\beta_1}+A_{C22}R^{\beta_2}=\frac{R_H}{r}+A_{C32}R^{\beta_2} \tag{8}$$

and its smooth-pasting condition expressed as:

$$\frac{R}{\delta}+\beta_1 A_{C21}R^{\beta_1}+\beta_2 A_{C22}R^{\beta_2}=\beta_2 A_{C32}R^{\beta_2} \tag{9}$$

both hold when evaluated at $R=R_H$. This reveals that:

$$A_{C11}=\left[\frac{R_H}{R_H^{\beta_1}}-\frac{R_L}{R_L^{\beta_1}}\right]\times\frac{\left(r\beta_2-r-\delta\beta_2\right)}{\left(\beta_1-\beta_2\right)r\delta}>0,\ A_{C21}=\frac{R_H\left(r\beta_2-r-\delta\beta_2\right)}{R_H^{\beta_1}\left(\beta_1-\beta_2\right)r\delta}<0,$$
$$A_{C22}=\frac{-R_L\left(r\beta_1-r-\delta\beta_1\right)}{R_L^{\beta_2}\left(\beta_1-\beta_2\right)r\delta}>0,\ A_{C32}=\left[\frac{R_H}{R_H^{\beta_2}}-\frac{R_L}{R_L^{\beta_2}}\right]\times\frac{\left(r\beta_1-r-\delta\beta_1\right)}{\left(\beta_1-\beta_2\right)r\delta}<0. \tag{10}$$

14.3 Revenue Floor Model

The additional subscript f indicates a model with only a floor. From (5) the active project valuation function becomes:

$$V_{Cf}\left(R\right)=\begin{cases}\dfrac{R_L}{r}+A_{Cf11}R^{\beta_1} & \text{for R} \le R_L\\ \dfrac{R}{\delta}+A_{Cf22}R^{\beta_2} & \text{for R}_L \le R,\end{cases} \tag{11}$$

with:
$$A_{Cf11}=\frac{-R_L\left(r\beta_2-r-\delta\beta_2\right)}{R_L^{\beta_1}\left(\beta_1-\beta_2\right)r\delta}\ge 0,\ A_{Cf22}=\frac{-R_L\left(r\beta_1-r-\delta\beta_1\right)}{R_L^{\beta_2}\left(\beta_1-\beta_2\right)r\delta}\ge 0. \tag{12}$$

14.4 Ceiling Only Model

The additional subscript c indicates a model with only a ceiling. The active project valuation function is:

$$V_{Cc}\left(R\right)=\begin{cases}\dfrac{R}{\delta}+A_{Cc21}R^{\beta_1} & \text{for } R<R_H\\ \dfrac{R_H}{r}+A_{Cc32}R^{\beta_2} & \text{for } R_H \le R,\end{cases} \tag{13}$$

with:

$$A_{Cc21}=\frac{R_H}{R_H^{\beta_1}}\frac{r\beta_2-r-\delta\beta_2}{r\left(\beta_1-\beta_2\right)\delta}\le 0,\quad A_{Cc32}=\frac{R_H}{R_H^{\beta_2}}\frac{r\beta_1-r-\delta\beta_1}{r\left(\beta_1-\beta_2\right)\delta}\le 0\,. \tag{14}$$

14.5 Numerical Illustrations

Suppose the current net revenue is 10 with a volatility of 20%, no operating costs, and r=δ=4%. Figure 1 shows that if the owner has obtained a guarantee in perpetuity at R_L=5, with a ceiling R_H=15, the ROV of operating such a perpetual activity is (5), while without a collar the present value is VC PV=R/δ =250 (cell B12). With a collar, the VC=ROV=250-55.56 call plus 20.83 put=215.28 (cell B11). Figure 1

	A	B	C	D
1			**ACTIVE PPP WITH COLLAR**	
2	**INPUT**			EQ
3	R	10.00		
4				
5	σ	0.20		
6	r	0.04		
7	δ	0.04		
8	R_L	**5**		
9	R_H	**15**		
10	**OUTPUT**			
11	VC	**215.2778**	**IF(B3<$B$8,$B$8/B6+B16,IF(B3>$B$9,$B$9/B6+B19,B3/B7+B17+B18))**	5
12	VC PV	**250.0000**	**IF(B3<B8,B8/B6,IF(B3>B9,B9/B6,B3/B7))**	5
13	R/δ	250.0000	B3/B7	
14	β_1	2.0000	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
15	β2	-1.0000	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
16	AC11*R^β1	**111.1111**	**B21*(B3^B14)**	
17	AC21*R^β1	**-55.5556**	**B22*(B3^B14)**	
18	AC22*R^β2	**20.8333**	**B23*(B3^B15)**	
19	AC32*R^β2	**-166.6667**	**B24*(B3^B15)**	
20	VC	215.2778	B12+B17+B18	
21	AC11	1.1111	(B9/(B9^B14)-B8/(B8^B14))*(B25/B27)	10
22	AC21	-0.5556	(B9/(B9^B14))*(B25/B27)	10
23	AC22	208.3333	(-B8/(B8^B15))*(B26/B27)	10
24	AC32	-1666.67	(B9/(B9^B15)-B8/(B8^B15))*(B26/B27)	10
25	[ ]	-0.0400	(B6*B15-B6-B7*B15)	10
26	( )	-0.0400	(B6*B14-B6-B7*B14)	10
27	{ }	0.0048	(B14-B15)*B6*B7	10
28				
29	ODE	0.0000	0.5*(B5^2)*(B3^2)*B31+(B6-B7)*B3*B30-B6*B11+MIN(MAX(B8,B3),B9)	2
30	VC Δ	11.8056		
31	VC Γ	-0.6944		
32	VC Δ	IF(B3<B8,B14*B21*(B3^(B14-1)),IF(B3>B9,B15*B24*(B3^(B15-1)),1/B7+B14*B22*(B3^(B14-1))+B15*B23*(B3^(B15-1))))		
33	VC Γ	IF(B3<B8,B14*(B14-1)*B21*(B3^(B14-2)),IF(B3>B9,B15*(B15-1)*B24*(B3^(B15-2)),B14*(B14-1)*B22*(B3^(B14-2))+B15*(B15-1)*B23*(B3^(B15-2))))		
34	VC Δ			
35	IF(B3<B8)	22.2222	B14*B21*(B3^(B14-1))	
36	IF(B8<B3<B9)	11.8056	1/B7+B14*B22*(B3^(B14-1))+B15*B23*(B3^(B15-1))	
37			Decomposition	
38	IF(B8<B3<B9)	25.0000	1/B7	
39	IF(B8<B3<B9)	-11.1111	B14*B22*(B3^(B14-1))	
40	IF(B8<B3<B9)	-2.0833	B15*B23*(B3^(B15-1))	
41	IF(B9<B3)	16.6667	B15*B24*(B3^(B15-1))	

With only a floor guarantee at R_L=5 using (11), the VC_f=ROV=250+20.83=270.83 illustrated in Figure 2.

Figure 2

	A	B	C	D
1			**ACTIVE PPP WITH FLOOR ONLY**	
2	**INPUT**			EQ
3	R	10.00		
4				
5	σ	0.20		
6	r	0.04		
7	δ	0.04		
8	R_L	**5**		
9				
10	**OUTPUT**			
11	VCf	**270.8333**	**IF(B3<B8,B8/B6+B16,B3/B7+B18)**	11
12	VCf PV	**250.0000**	**IF(B3<B8,B8/B6,B3/B7)**	11
13	R/δ	250.0000	B3/B7	
14	β_1	2.0000	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
15	β2	-1.0000	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
16	ACf11*R^β1	**166.6667**	**B21*(B3^B14)**	
17				
18	ACf22*R^β2	**20.8333**	**B23*(B3^B15)**	
19				
20				
21	ACf11	1.6667	(-B8/(B8^B14))*(B25/B27)	12
22				
23	ACf22	208.3333	(-B8/(B8^B15))*(B26/B27)	12
24				
25	[ ]	-0.0400	(B6*B15-B6-B7*B15)	
26	( )	-0.0400	(B6*B14-B6-B7*B14)	
27	{ }	0.0048	(B14-B15)*B6*B7	
28				
29	ODE	0.0000	0.5*(B5^2)*(B3^2)*B31+(B6-B7)*B3*B30-B6*B11+MIN(MAX(B8,B3))	
30	VC Δ	22.9167	IF(B3<B8,1/B6+B14*B21*(B3^(B14-1)),1/B7+B15*B23*(B3^(B15-1)))	
31	VC Γ	0.4167	IF(B3<B8,(B14-1)*B14*B21*(B3^(B14-2)),(B15-1)*B15*B23*(B3^(B15-2)))	
32	VC Δ			
33	IF(B3<B8)	33.3333	B14*B21*(B3^(B14-1))	
34	IF(B8<B3	22.9167	1/B7+B15*B23*(B3^(B15-1))	
35			Decomposition	
36	IF(B8<B3	25.0000	1/B7	
37	IF(B8<B3	-2.0833	B15*B23*(B3^(B15-1))	

With only a ceiling, the VC_c=ROV=194.44, which is 250 less 55.56. These results are very sensitive in changes in most of the parameter values.

Figure 3

	A	B	C	D
1			**ACTIVE PPP WITH CEILING ONLY**	
2	**INPUT**			EQ
3	R	10.00		
4				
5	σ	0.20		
6	r	0.04		
7	δ	0.04		
8				
9	R_H	**15.0000**		
10	**OUTPUT**			
11	VCc	**194.4444**	**IF(B3<B9,B3/B7+B17,B9/B6+B19)**	13
12				
13	R/δ	250.0000	B3/B7	
14	β_1	2.0000	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
15	β2	-1.0000	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4
16				
17	ACc21*R^β1	**-55.5556**	**B22*(B3^B14)**	
18				
19	ACc32*R^β2	**-187.5000**	**B24*(B3^B15)**	
20				
21				
22	ACc21	-0.5556	(B9/(B9^B14))*(B25/B27)	14
23				
24	ACc32	-1875.00	(B9/(B9^B15))*(B26/B27)	14
25	[ ]	-0.0400	(B6*B15-B6-B7*B15)	
26	( )	-0.0400	(B6*B14-B6-B7*B14)	
27	{ }	0.0048	(B14-B15)*B6*B7	
28				
29	ODE	0.0000	0.5*(B5^2)*(B3^2)*B31+(B6-B7)*B3*B30-B6*B11+MAX(MIN(B3,B9))	
30	VC Δ	13.8889	IF(B3<B9,1/B7+B14*B22*(B3^(B14-1)),1/B6+B15*B24*(B3^(B15-1)))	
31	VC Γ	-1.1111	IF(B3<B9,(B14-1)*B14*B22*(B3^(B14-2)),(B15-1)*B15*B24*(B3^(B15-2)))	
32	VC Δ			
33	IF(B3<B9)	13.8889	1/B7+B14*B22*(B3^(B14-1))	
34			Decomposition	
35	IF(B3<B9)	25.0000	1/B7	
36	IF(B3<B9)	-11.1111	B14*B22*(B3^(B14-1))	
37	IF(B9<B3	18.7500	B15*B24*(B3^(B15-1))	

Note that the differential equation (2) is solved (B29), calculating the ROV deltas and gammas in B30 and B31. Further discussion of ROV "Greeks" is in the Appendix.

14. 6 Investment Option with a Collar

The value of a perpetual opportunity to invest in a project with a stochastic revenue and fixed investment cost is a fundamental equation in the literature of real options. The without-collar optimal price threshold level triggering investment $\hat{R}_0$ is:

$$\hat{R}_0 = \frac{\beta_1}{\beta_1 - 1}\delta K \ , \tag{15}$$

and the value function is:

$$F_0\left(R\right) = \begin{cases} = \dfrac{K}{\beta_1 - 1}\left(\dfrac{R}{\hat{R}_0}\right)^{\beta_1} & \text{for } R < \hat{R}_0 \\ = \dfrac{R}{\delta} - K & \text{for R} \geq \hat{R}_0, \end{cases} \tag{16}$$

with:

$$A_0 = \frac{K\hat{R}_0^{-\beta_1}}{\beta_1 - 1}. \tag{17}$$

The with-collar optimal price threshold $\hat{R}_C$ triggering an investment lies between the floor and cap limits, $R_L \leq \hat{R}_C \leq R_H$. When $R_L \leq \hat{R}_C \leq R_H$, the optimal solution is obtained from equating the investment option value with the active project net value at the threshold $R = \hat{R}_C$. The optimal solution is determined from both the value-matching relationship:

$$A_{C0}R^{\beta_1} = \frac{R}{\delta} + A_{C21}R^{\beta_1} + A_{C22}R^{\beta_2} - K \tag{18}$$

and its smooth-pasting condition expressed as:

$$\beta_1 A_{C0}R^{\beta_1} = \frac{R}{\delta} + \beta_1 A_{C21}R^{\beta_1} + \beta_2 A_{C22}R^{\beta_2} \tag{19}$$

when evaluated for $R = \hat{R}_C$. This reveals that:

$$\frac{\hat{R}_C}{\delta} = \frac{\beta_1}{\beta_1 - 1}K - \frac{\beta_1 - \beta_2}{\beta_1 - 1}A_{C22}\hat{R}_C^{\beta_2} \tag{20}$$

$$A_{C0} = \frac{1}{\beta_1 - \beta_2}\left[\left(1-\beta_2\right)\frac{\hat{R}_C}{\delta} + \beta_2 K\right]\hat{R}_C^{-\beta_1} + A_{C21}. \quad (21)$$

The absence of a closed-form solution requires $\hat{R}_C$ to be solved numerically from (20), and A_{C0} from (21). The investment option value $F_{C0}\left(R\right)$ for the project, is:

$$F_{C0}\left(R\right) = \begin{cases} A_{C0}R^{\beta_1} & \text{for } R < \hat{R}_C \\ \frac{R}{\delta} - K + A_{C21}R^{\beta_1} + A_{C22}R^{\beta_2} & \text{for } \hat{R}_C \le R < R_H, \end{cases} \quad (22)$$

Figure 4

	A	B	C	D
1	**INVESTMENT OPPORTUNITY FOR A PPP WITH A COLLAR OPTION**			
2	**INPUT**			EQ
3	R	6.00		
4	K	100.00		
5	σ	0.25		
6	r	0.04		
7	δ	0.04		
8	RL	**4**		
9	RH	**10**		
10	**OUTPUT**			
11	ROV CALL	**61.8978**	IF(B3<B13,((B4/(B14-1))*(B3/B13)^B14),B12)	16
12	R/δ-K	50.0000	MAX(B3/B7-B4,0)	
13	R^	9.4279	(B14/(B14-1))*B4*B7	15
14	β_1	1.7369	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/	4
15	A0	2.7547	(B4*(B13^-B14))/(B14-1)	17
16				
17	ROV COLLAR	**38.3688**	IF(B3<B20,B21*(B3^B14),B3/B7-B4+B23*(B3^B14)+B24*(	22
18	β2	-0.7369	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(	4
19	FIND R^	0.0000	B20/B7-(B14/(B14-1))*B4+((B14-B18)/(B14-1))*B24*(B20^	20
20	R^	4.0000	SOLVER Set B19=0, changing B20	
21	AC0	1.7862	(1/(B14-B18))*((1-B18)*(B20/B7)+B18*B4)*(B20^-B14)+B2	21
22				
23	AC21	-1.8520	($B$9/($B$9^B14))*(B26/B28)	10
24	AC22	112.2797	(-$B$8/($B$8^B18))*(B27/B28)	10
25				
26	[ ]	-0.0400	(B6*B18-B6-B7*B18)	10
27	( )	-0.0400	(B6*B14-B6-B7*B14)	10
28	{ }	0.0040	(B14-B18)*B6*B7	10
29				
30	AC21*R^β1	**-41.6129**	**B23*(B3^B14)**	RL<R<RH
31	AC22*R^β2	**29.9818**	**B24*(B3^B18)**	
32	ROV COLLAR	38.3688	B12+B30+B31	

As shown in Figure 4, the threshold $\hat{R}_C$ depends only on the floor R_L through A_{C22}, but not on the ceiling R_H. Adjusting the ceiling of the collar has no material impact on the threshold, so the timing decision is affected by the losses foregone by having a floor but not by the gains sacrificed by having a ceiling. Since A_{C22} is non-negative, the with-collar threshold $\hat{R}_C$ is always no greater than the without-collar threshold $\hat{R}_0$, and an increase in the floor produces an earlier exercise due to the reduced threshold level.

Figure 4 shows that with a floor of 4 and ceiling of 10, and the other parameter values, the option coefficients A_{C21} and A_{C22} are -1.8520 and 112.2797, so the ROV COLLAR is 38.4 when $R_L<R<R_H$, less than the ROV without collar 61.9. Cell B31 shows that the ROV (COLLAR)=NPV(50)+ PUT(29.98)-CALL (41.61)=38.37. An investment opportunity with only a put is worth 50+29.98=79.98, and with only a written call 50-41.61=8.39. These values are also very sensitive to changes in the parameter values, as shown in the Appendix.

EXERCISE 14.1

Carlos Azevedo owns a solar plant, with an annual revenue R=€ 2, but the generous Portuguese government has guaranteed a revenue of € 4 per annum. If r=.04, electricity δ=.04, σ=20%, should Carlos try to sell this plant for €100, if A_{Cf11}=2.08, A_{Cf22}=133.33?

$$\beta_1,\beta_2=\left(\tfrac{1}{2}-\frac{r-\delta}{\sigma^2}\right)\pm\sqrt{\left(\tfrac{1}{2}-\frac{r-\delta}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}$$

$$V_{Cf}\left(R\right)=\begin{cases}\dfrac{R_L}{r}+A_{Cf11}R^{\beta_1} & \text{for R} \le R_L\\ \dfrac{R}{\delta}+A_{Cf22}R^{\beta_2} & \text{for R}_L \le R,\end{cases}$$

EXERCISE 14.2

Generous Carlos Azevedo believes his solar plant with an annual revenue R of € 4, might be sold with all the revenues over € 15 per annum reserved for the Universidad de Minho. If r=.04, electricity δ=.04, σ=20%, should Carlos try to sell this plant for €100, if A_{Cc21}=-.5556, A_{Cc32}=-1875?

$$\beta_1,\beta_2=\left(\tfrac{1}{2}-\frac{r-\delta}{\sigma^2}\right)\pm\sqrt{\left(\tfrac{1}{2}-\frac{r-\delta}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}$$

$$V_{Cc}\left(R\right)=\begin{cases}\dfrac{R}{\delta}+A_{Cc21}R^{\beta_1} & \text{for } R<R_H\\ \dfrac{R_H}{r}+A_{Cc32}R^{\beta_2} & \text{for } R_H\le R,\end{cases}$$

EXERCISE 14.3

Clever Carlos Azevedo owning a solar plant with an annual revenue of R=€4 has obtained from his friend the Minister of Energy a minimum revenue guarantee of € 4 per annum, but wants all revenues over 1000 p.a. to go to the Universidad of Minho. If r=.04, electricity δ =.04, σ=20%, should Carlos try to sell this plant for €100, if A_{C21}=-.0083, A_{C22}=133.33?

$$\beta_1,\beta_2=\left(\tfrac{1}{2}-\frac{r-\delta}{\sigma^2}\right)\pm\sqrt{\left(\tfrac{1}{2}-\frac{r-\delta}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}$$

$$V_C\left(R\right)=\begin{cases}\dfrac{R_L}{r}+A_{C11}R^{\beta_1} & \text{for } R<R_L\\ \dfrac{R}{\delta}+A_{C21}R^{\beta_1}+A_{C22}R^{\beta_2} & \text{for } R_L\le R<R_H\\ \dfrac{R_H}{r}+A_{C32}R^{\beta_2} & \text{for } R_H\le R.\end{cases}$$

PROBLEM 14.4

Carlos Azevedo owns a solar plant, with a revenue of €10, but a friendly Portuguese government has guaranteed a revenue of €6 per annum. If r=.04, electricity δ =.04, σ=25%, should Carlos try to sell this plant for €300?

$$\beta_1,\beta_2=\left(\tfrac{1}{2}-\frac{r-\delta}{\sigma^2}\right)\pm\sqrt{\left(\tfrac{1}{2}-\frac{r-\delta}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}$$

$$V_{Cf}\left(R\right)=\begin{cases}\dfrac{R_L}{r}+A_{Cf11}R^{\beta_1} & \text{for R}\le R_L\\ \dfrac{R}{\delta}+A_{Cf22}R^{\beta_2} & \text{for R}_L\le R,\end{cases}$$

PROBLEM 14.5

Carlos Azevedo owns a solar plant, with a revenue of €10, but a mean Portuguese government has guaranteed a revenue of €2 per annum, and demanded a ceiling of €12. If r=.04, electricity δ =.04, σ=25%, should Carlos try to sell this plant for €150?

$$\beta_1, \beta_2 = \left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$$

$$V_{Cf}\left(R\right) = \begin{cases} \dfrac{R_L}{r} + A_{Cf11} R^{\beta_1} & \text{for R} \le R_L \\ \dfrac{R}{\delta} + A_{Cf22} R^{\beta_2} & \text{for R}_L \le R, \end{cases} \qquad 6$$

PROBLEM 14.6

Carlos Azevedo has an opportunity to invest in a monopoly for profit university with an expected revenue of €4, with an investment cost of €100. A smart Portuguese government has guaranteed a revenue of €4 per annum, and demanded a ceiling of €10. If r=.04, asset yield δ =.04, σ=25%, should Carlos buy this perpetual concession for €20? See (20) (21) (22) above.

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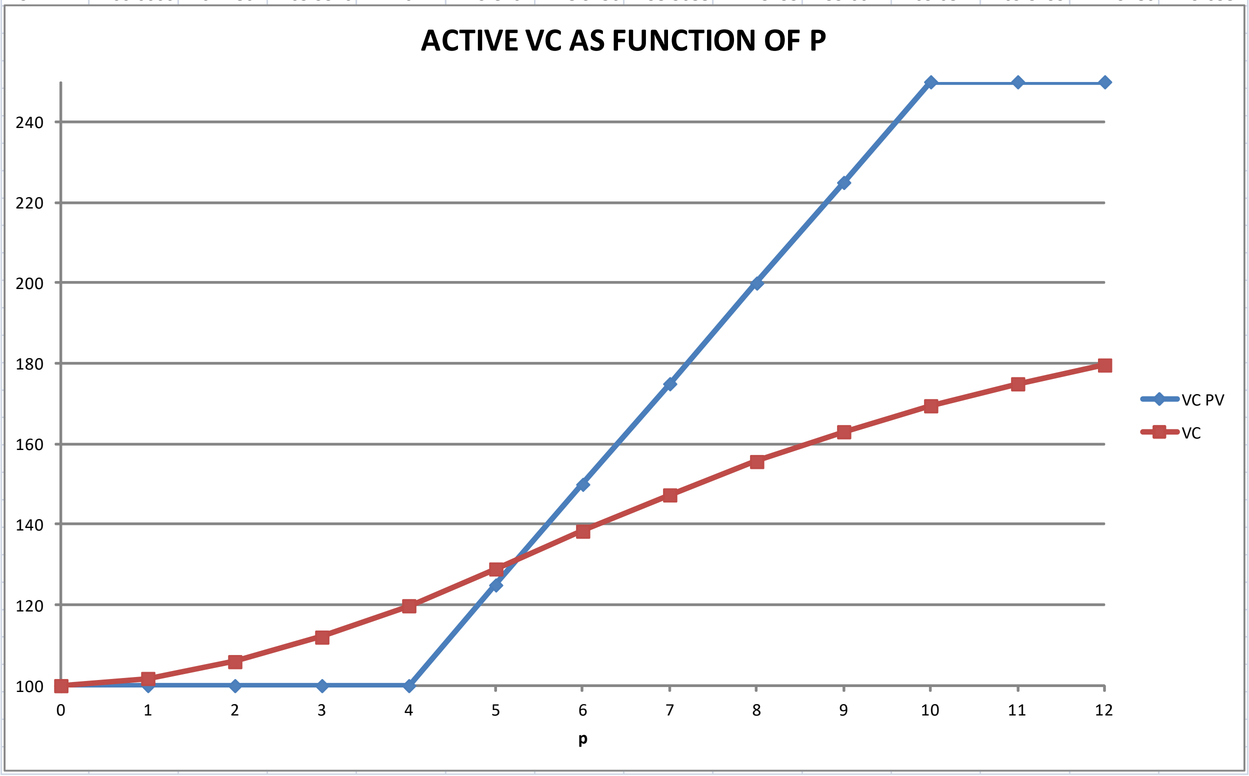
APPENDIX

Figure A2

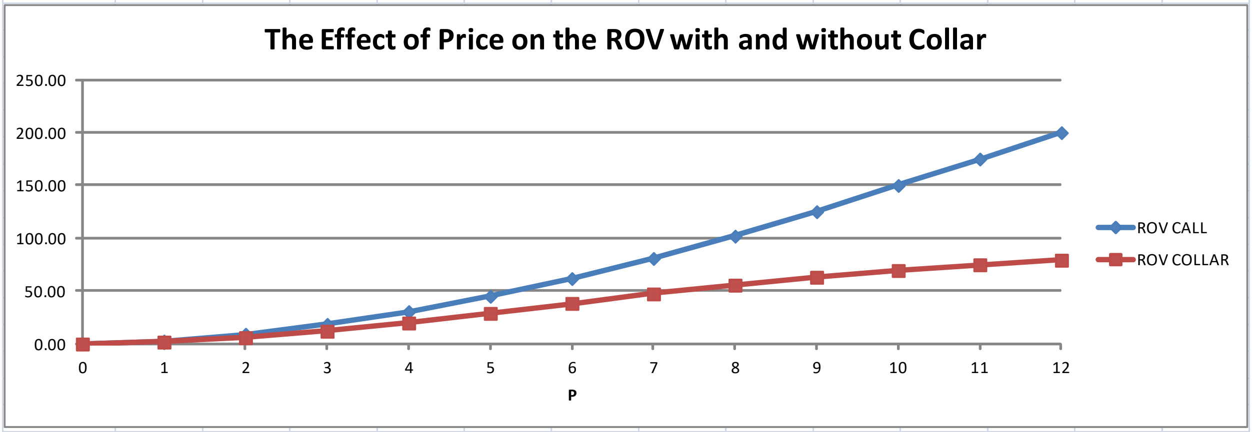
P	0	1	2	3	4	5	6	7	8	9	10	11	12
VC PV	100.0000	100.0000	100.0000	100.0000	100.0000	125.0000	150.0000	175.0000	200.0000	225.0000	250.0000	250.0000	250.0000
VC	100.0000	101.7862	105.9540	112.0411	119.8462	128.9756	138.3688	147.3733	155.6677	163.0814	169.5199	174.9786	179.6381



In Figure A2, past the floor price of P_L=4, the difference between the VC PV and the VC consists of a long position in a put option (should P go below 4) and a short position in a call option (should P rise above 10=P_H). If P=6, the net value of the put and call is negative, so the VC PV exceeds the VC. The (VC PV – VC) spread increases as P increases up to 10, the ceiling price.

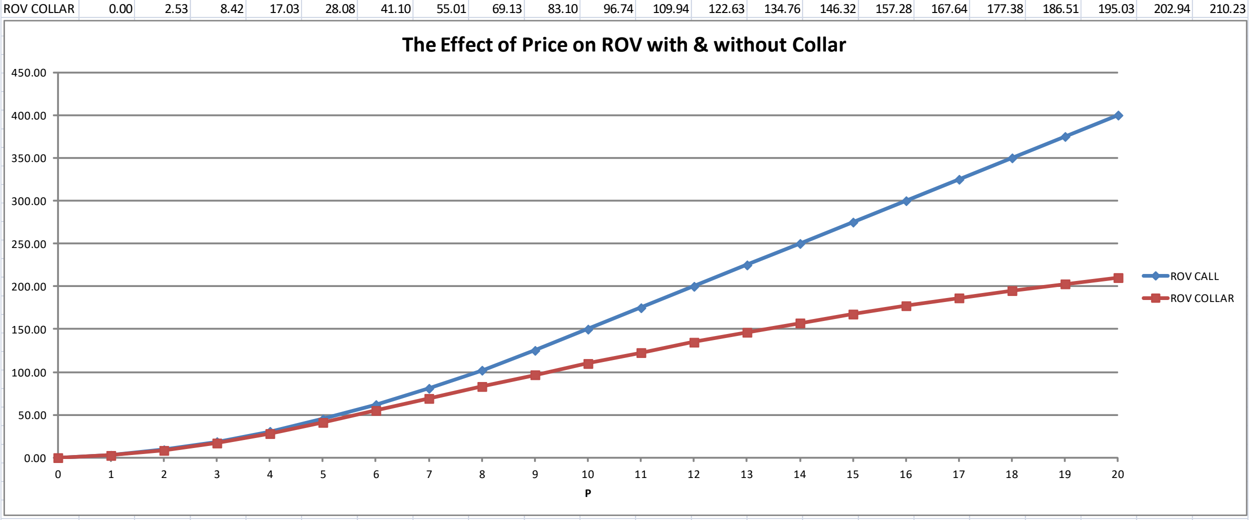
Figure A3

P	0	1	2	3	4	5	6	7	8	9	10	11	12
ROV CALL	0.00	2.75	9.18	18.57	30.61	45.10	61.90	80.90	102.02	125.18	150.00	175.00	200.00
ROV COLLAR	0.00	1.79	5.95	12.04	19.85	28.98	38.37	47.37	55.67	63.08	69.52	74.93	79.28



In Figure A3, the ROV Collar (P_L=4, P_H=10) always has a lower value than a standard ROV without a collar, since there is no upper limit to the investment profit, and the investment opportunity is an option, not yet a commitment. Figure A4

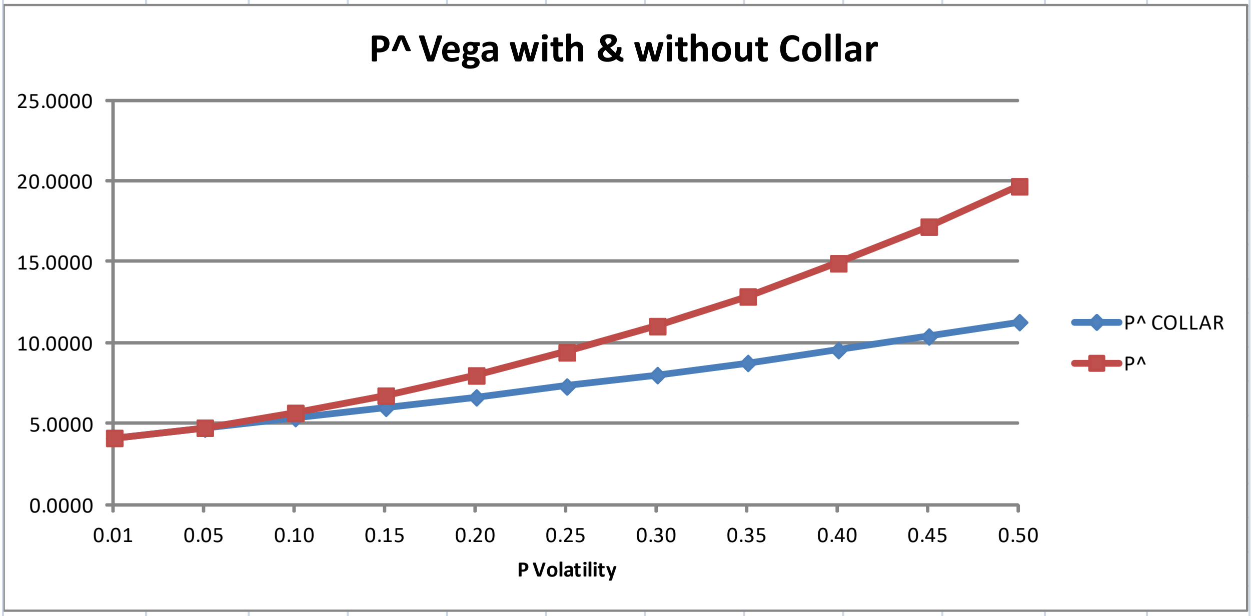
P	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ROV CALL	0.00	2.75	9.18	18.57	30.61	45.10	61.90	80.90	102.02	125.18	150.00	175.00	200.00	225.00	250.00	275.00	300.00	325.00	350.00	375.00	400.00
ROV COLLAR	0.00	2.53	8.42	17.03	28.08	41.10	55.01	69.13	83.10	96.74	109.94	122.63	134.76	146.32	157.28	167.64	177.38	186.51	195.03	202.94	210.23

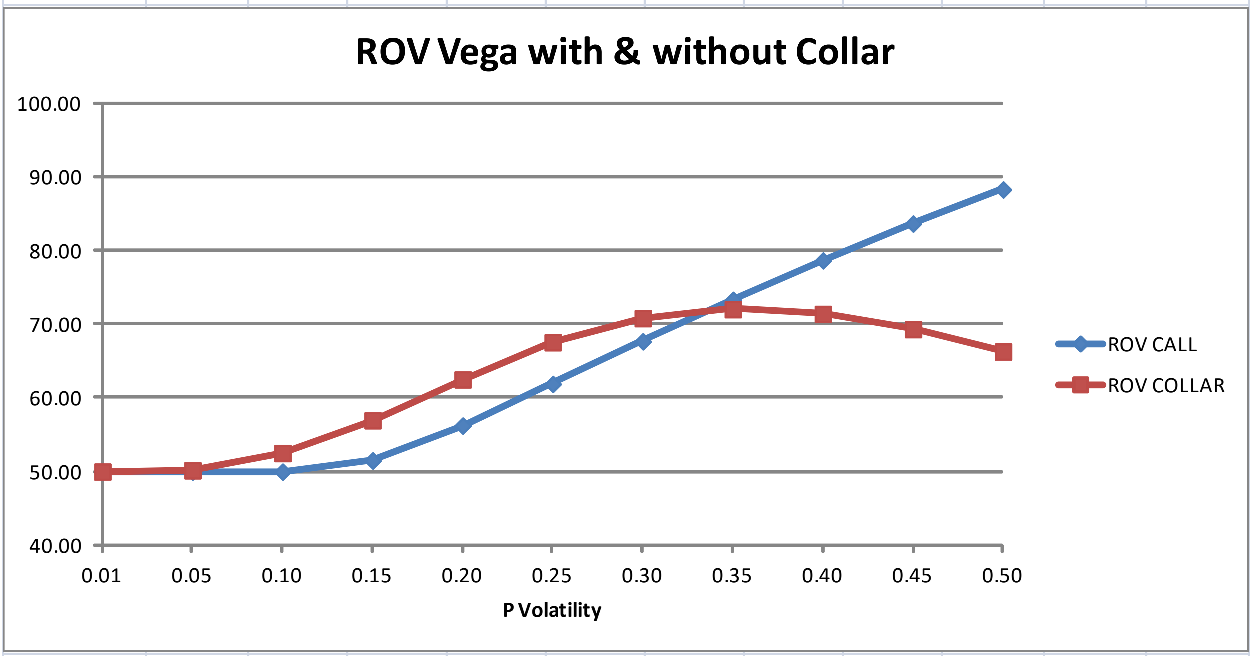


PL **4** PH **20**

In Figure A4, the ROV Collar with a higher price ceiling, in this case P_H=20, is more valuable than with the previous ceiling of P_H=10, and the spread between the ROV with and without collar increases as P approaches P_H. Figure A5

σ	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ROV CALL	50.00	50.00	50.00	51.51	56.25	61.90	67.69	73.32	78.66	83.65	88.28
ROV COLLAR	50.00	50.18	52.52	56.97	62.51	67.55	70.82	72.03	71.40	69.34	66.30
P^ COLLAR	4.1439	4.7168	5.3681	6.0008	6.6458	7.3178	8.0254	8.7739	9.5670	10.4074	11.2970
P^	4.1439	4.7724	5.6861	6.7571	8.0000	9.4279	11.0523	12.8831	14.9282	17.1945	19.6873





P	2		PL	3						PH	500

What is the affect of increasing volatility of the primary underlying factor on the threshold that justifies immediate investment, and also on the ROV (the so-called "vega"). Naturally the price threshold increases with the increased of expected price volatility shown in Figure A5 (P=2, P_L=3, P_H=500), so a government seeking early investment might consider imposing a collar in a volatile price environment. The ROV without a collar increases almost linearly with increases in the price volatility, but the ROV with a collar has a different pattern. From a low volatility environment, the ROV + Collar increases, but eventually at high expected volatilities the vega almost becomes negative, due to the increase in the value of the written call option.

ACTIVE MODEL GREEKS

Samuelson (1965) established that a call option could replicated by shorting DELTA of the underlying asset, and adjusting this position throughout time as the asset changes. Similarly, the floor or ceiling options could be hedged by dynamic positions equal to the floor FDELTA or CDELTA, that is the change in the call or put option value as the floor or ceiling changes. These DELTAs are easy to calculate, realising that the DELTA will depend on the Regimes that is whether $R<R_L$, or $R_L<R<R_H$, or $R_H<R$.

COLLAR DELTA

$$\Delta V_C\left(R\right)=\begin{cases}\beta_1 A_{C11} R^{(\beta_1-1)} & \text{for } \mathrm{R} < R_L \\ \dfrac{1}{\delta}+\beta_1 A_{C21} R^{(\beta_1-1)}+\beta_2 A_{C22} R^{(\beta_2-1)} & \text{for } \mathrm{R}_L \le R < R_H \\ \beta_2 A_{C32} R^{(\beta_2-1)} & \text{for } \mathrm{R}_H \le R.\end{cases} \tag{A1}$$

FLOOR DELTA

$$\Delta V_{Cf}\left(R\right)=\begin{cases}\beta_1 A_{Cf11} R^{(\beta_1-1)} & \text{for } \mathrm{R} \le R_L \\ \dfrac{1}{\delta}+\beta_2 A_{Cf22} R^{(\beta_2-1)} & \text{for } \mathrm{R}_L \le R,\end{cases} \tag{A2}$$

CEILING DELTA

$$\Delta V_{Cc}\left(R\right)=\begin{cases}\dfrac{1}{\delta}+\beta_1 A_{Cc21} R^{(\beta_1-1)} & \text{for } R < R_H \\ \beta_2 A_{Cc32} R^{(\beta_2-1)} & \text{for } R_H \le R,\end{cases} \tag{A3}$$